NUMERICAL VALIDATION AND APPLICATION OF THE NEUBER-FORMULA IN FEA-ANALYSIS

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SUMMARY

For the simulation of the durability and life estimation of cyclic loaded parts, simulation models which consider material plasticity and damage effects such as the local strain concepts are state of the art. Typically light weight structures are dimensioned in a way that limited local yielding is allowed. Traditional nonlinear FEA analysis simulating the local material plasticity are still very resource intensive, yet fatigue and life endurance simulations commonly need stress and strain results for various different load levels, making such an analysis expensive. In order to reduce the number of nonlinear simulation results, approximation techniques based on the Neuber formula which estimate the plastic stress-strain state from linear analysis runs are utilized in commercial fatigue simulation software such as "NEi Fatigue/Winlife", FE-Fatigue or "MSC Fatigue", to name a few.

To validate the Neuber approach, this paper compares notched test specimen equipped with strain gages to the results of a finite element analysis with an elastic-plastic material model and different Neuber-based approximations.

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1: Introduction

In the past decades, technical progress and the increasing utilization of Finite Element simulations lead to lighter components with their shape adapted to efficiently bear the applied loads. A further trend in structural parts optimisation is to build them not for an infinite life endurance, but rather for the expected service life, including a safety margin. The general goal is further weight and material reduction in order to increase competitiveness, while on the other hand gaining knowledge about and improving of reliability and safety over the product life cycle.

While some parts have to sustain a constant cyclic loading during their entire life, many parts are loaded with a random load range over time. Load spikes commonly lead to conditions were local yielding is observed. The simulation of a component's fatigue behaviour therefore must include nonlinear material effects. While the solution of finite element simulations with nonlinear materials has been state of the art for several years, it still is by magnitudes more expensive then a linear static solution. For a fatigue analysis typically the FEA results for various load levels are theoretically required, multiplying the analysis expenses into regions where they would become often prohibitive expensive. However, in the early 1960s Heinz Neuber introduced a method to calculate strains and stresses exceeding the material yield point based on the nominal stress and notch concentration factors [1].

With the application of Finite Element Analysis, notch concentration factors are being inherently considered. The general Neuber procedure of extrapolating linear stresses into the plastic material region can thus be applied to arbitrary geometries.

2: The Neuber Formula

Parts made of ductile material can be designed economically very efficient if they are not secured against yielding but allowing a limited amount of plastic deformation. The components dimensioning requires the knowledge of its yield curve. In the following we will look at a punched flat specimen under tensile loading as an example for an actual part:

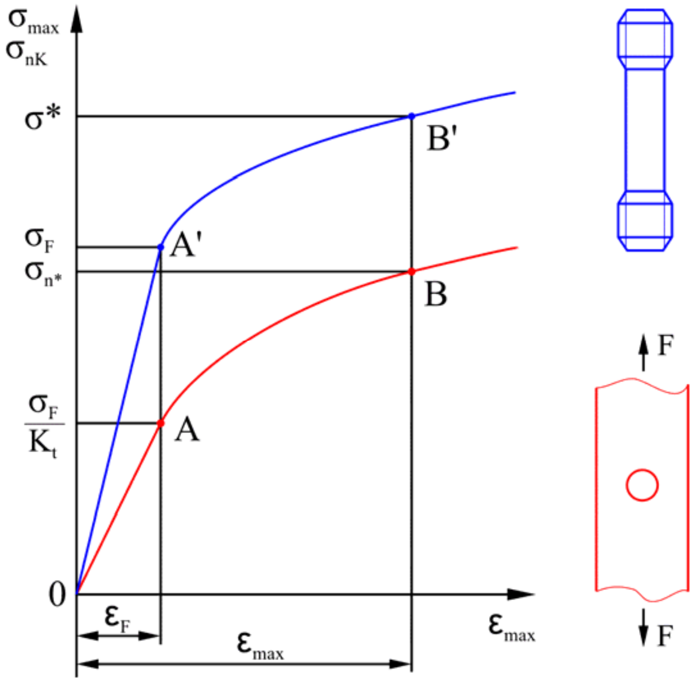

Figure 1: Stress-strain diagram of a tensile test specimen and nominal stress curve for the punched cross section.

Under uniaxial loading, yielding occurs when the stress in the notch reaches the yield strength σ_F, or respectively when the strain in the notch $\varepsilon_F = \sigma_F/E$. The yield point for the component (A) is determined by:

$$\sigma_{\max} = \sigma_{nF} \cdot K_t = \sigma_F \tag{1}$$

The nominal stress at the yield point hence is:

$$\sigma_{nF} = \frac{\sigma_F}{K_t} \tag{2}$$

The yield load is calculated by:

$$F_F = \sigma_{nF} \cdot A_k = \frac{\sigma_F}{K_t} \cdot A_k \tag{3}$$

In over-elastic loading, the proportionality between stress and strain respective load and strain is lost. Moreover, the notch concentration factor K_t becomes invalid. Because of the σ-ε relationship of the material, we can presume that the strains are over-proportional in the plastic range and the stresses increase under proportional compared to the linear section.

Because of the different stress-strain gradient in the plastic material range, the notch stress cannot be determined anymore by a concentration factor K_t. Instead we need different concentration factors for stresses and strains. The strain concentration factor K_ε is defined as the relation between the maximum strain ε_{max} in the notch and the nominal strain ε_n:

$$K_\varepsilon = \frac{\varepsilon_{\max}}{\varepsilon_n} \tag{4}$$

Analogical, the stress concentration factor K_σ can be expressed as the relation between the maximum stress σ_{max} in the notch and the nominal stress σ_n:

$$K_\sigma = \frac{\sigma_{\max}}{\sigma_n} \tag{5}$$

With uniaxial loading and elastic strains provided, the following relationship between nominal stress and nominal strain exists:

$$\varepsilon_n = \frac{\sigma_n}{E} \tag{6}$$

Between the three notch concentration factors, the following inequality applies:

$$K_\sigma \le K_t \le K_\varepsilon \tag{7}$$

Neuber showed on a shear loaded prism with a lateral groove that the stress- and strain concentration factors can be coupled through the relation:

$$K_\sigma \cdot K_\varepsilon = K_t^{\,2} \tag{8}$$

It was further shown that the equation (8) can be used to calculate component yield curves under different loading types. In Figure 2 the progression of the concentration factors according to equation (8) is plotted over the quotient $\varepsilon_{max}/\varepsilon_F$ for a punched plate:

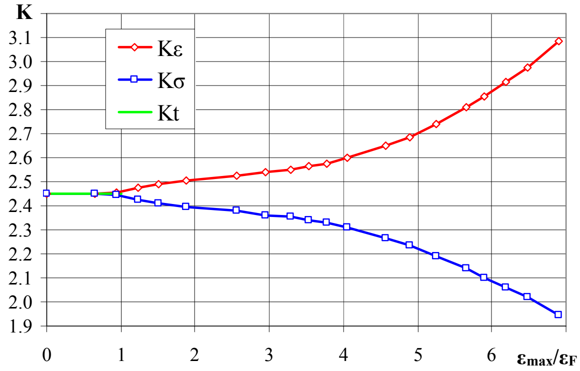

Figure 2: Stress and strain concentration factors for a punched plate according to equation (8)

From equations (4), (5) and (6) we can write:

$$\varepsilon_{\max} \cdot \sigma_{\max} = K_t^2 \cdot \frac{\sigma_n^2}{E} \tag{9}$$

On the left side of the equation we have the notch loading as product of its local stress and strain, the right side is defined by the notch geometry (K_t), the load (nominal stress σ_n) and the material (E). Further, we have the combination of σ and ε from the material stress-strain curve. In order to determine the actual stress and strain in the notch, we now only need to draw equation (9) into the stress-strain curve obtained from an unnotched tensile test. The intersection of the "Neuber hyperbola" from equation (9) with the stress-strain curve gives the actual local stress-strain state of the notch. It can be shown that equation (9) is also valid for the linear section, so no special consideration needs to be undertaken when applying this procedure.

Figure 3 shows the graphical determination of the notched stress-strain state utilizing the Neuber hyperbola within the material tensile curve.

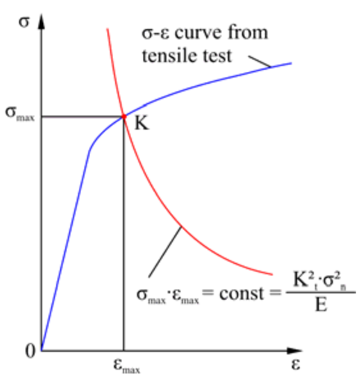

Figure 3: Determination of notch stress (point K) utilizing the Neuber hyperbola

In the design of a part, often the load for a given notch strain is of interest. In this case, and to compare these theoretical to the actual results later in this paper, we can write equation (9) in the form:

$$\sigma_n = \frac{\sqrt{E \cdot \varepsilon_{\max} \cdot \sigma(\varepsilon_{\max})}}{K_t} \tag{10}$$

The stress in the notch $\sigma(\varepsilon_{max})$ is retrieved from the σ-ε diagram. Equation (10) can be referred to as the yield curve of the part.

Dixon and Strannigan empirically found a modified approach to calculate the parts yield curve, which contains a correction expression in addition to the basic Neuber equation [2]:

$$\sigma_n = \frac{\sqrt{E \cdot \varepsilon_{\max} \cdot \sigma_{\max}}}{K_t \cdot \left(\dfrac{1+\mu_{el}}{1+\mu_{pl}}\right)} \quad (11), \qquad \text{with } \mu_{pl} = \frac{1}{2}\left(1-(1-2\mu_{el})\frac{S}{E}\right) \quad (12)$$

The secant modulus S is determined in for any given point on the stress strain curve through $S \equiv \dfrac{\sigma}{\varepsilon}$ (13)

A third variation of the Neuber equation is proposed by Sonsino [3], which determines the local strain by averaging the theoretical, linear extrapolated strain with the Neuber strain:

$$\varepsilon_{\max} = \frac{\varepsilon_{lin} + \varepsilon_{Neuber}}{2} \quad (14), \qquad \text{with } \varepsilon_{lin} = \frac{K_t \cdot \sigma_n}{E} \quad (15)$$

3: Experimental determination of stress and strain in a punched specimen

First, the material stress-strain curves were obtained on unnotched test specimen, Figure 4:

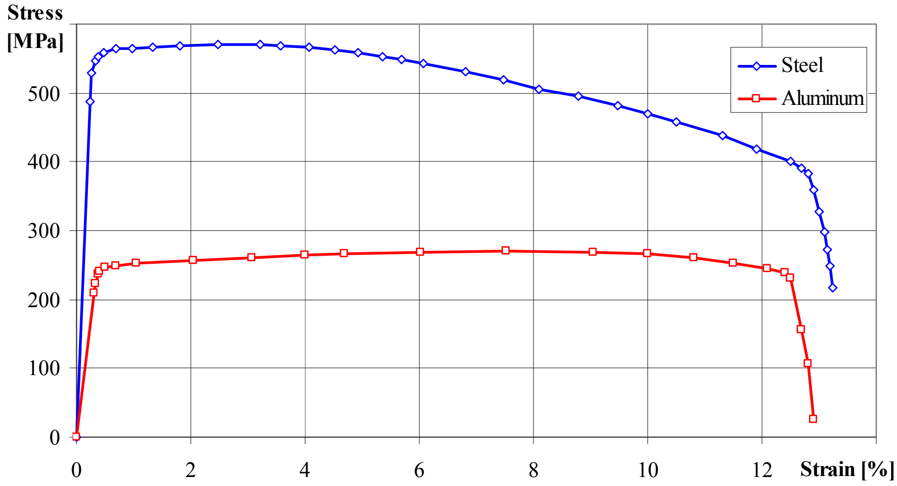

Figure 4: Stress-Strain Plot of the investigated test specimen

Then notched test specimens of the same materials (AlMgSi 1 and St-52) were prepared: The cross section was 8x40mm, and a central, 10mm diameter hole served as notch. Inside the hole two strain gages were applied as shown in Figure 5:

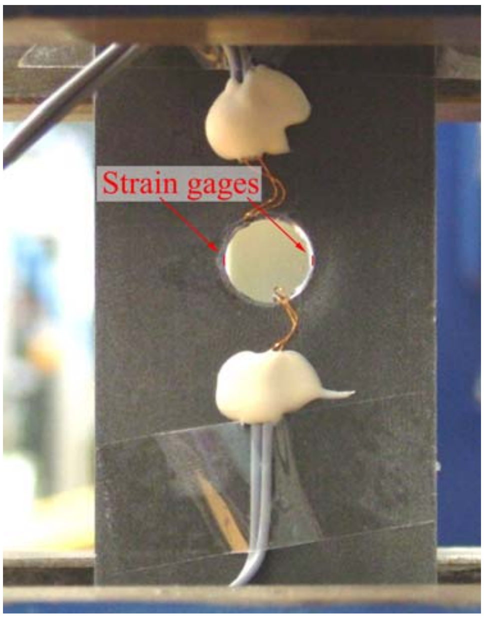

Figure 5: Punched test specimen equipped with strain gages

For the steel specimen, the two strain gages measured values differing ~10% from the mean value. Several reasons contributed to the difference:

1. One strain gage was not exactly positioned on the equator of the hole but offset by about 10°, measuring strains not on the hot spot but slightly lower values.
2. The other strain gage was not exactly positioned in the middle of the hole depth, but offset by ~0.5mm. While the test specimens were flat themselves, slight bending moments over the horizontal axis (Fig. 5) could have been introduced due to tolerances in the machine clamping, which would - due to the offset of the one strain gage - result in different readings.
3. Finally, the clamping itself may introduce slightly different load paths between the left and right side of the hole, resulting in the observed differences.

Because the source of the difference could not be pinpointed to only one of the mentioned reasons, the average value is subsequently considered.

The specimens were loaded and unloaded in several loops with increasing maximum loads, load and strain from the strain gages were recorded, Figures 6 and 7:

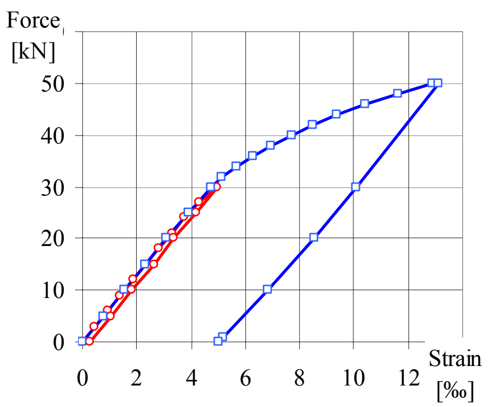

Figure 6: Aluminum specimen yield curve

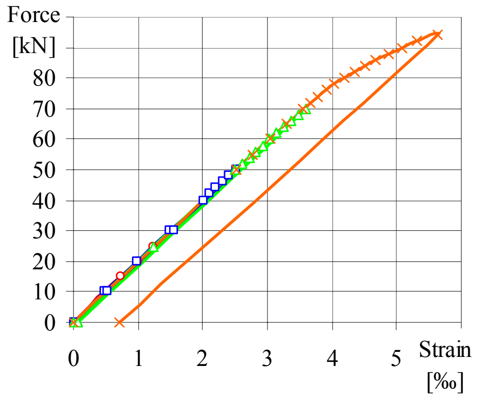

Figure 7: Steel specimen yield curve

4: Finite Element Analysis of the specimen

In order to being able to compare the experimental results to the Neuber formulae, we need to determine K_t. For the plate with a central hole, Roark [4] gives the empiric formula:

$$K_{tz} = 3 - 3.13 \cdot \left(\frac{2R}{B}\right) + 3.66 \cdot \left(\frac{2R}{B}\right)^2 - 1.53 \cdot \left(\frac{2R}{B}\right)^3 \tag{16}$$

With the Radius R=5mm and the specimen width of 40mm, equation (16) results to $\mathbf{K_{tz} = 2.42}$. The solution according to Roark was validated with a linear static solution for the specimen utilizing Noran Engineering NEiNastran version 8.3, Figure 8:

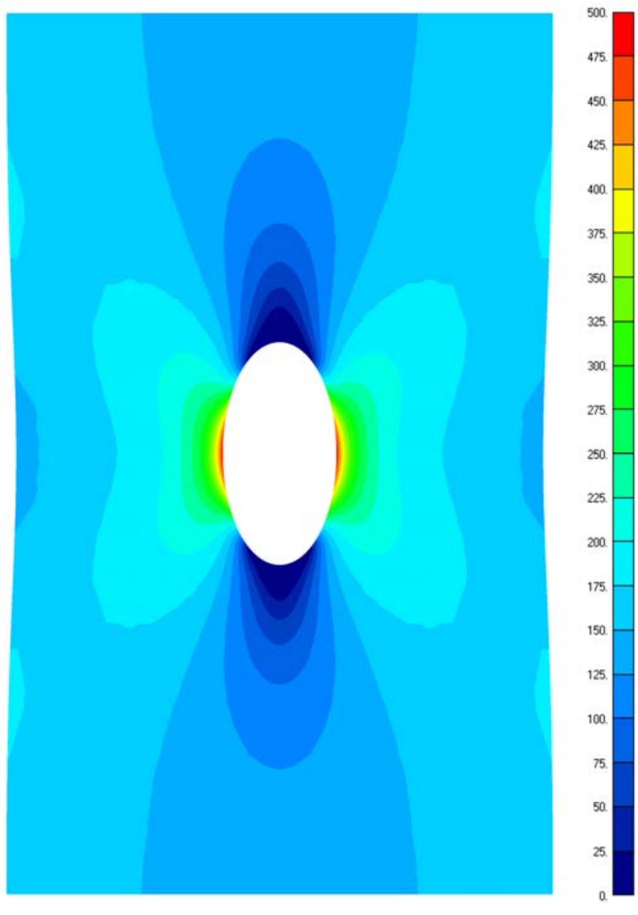

Figure 8: FE analysis to determine stress concentration factor

With a nominal tension in the remaining cross section of 200 MPa, the FE analysis returned 489.2 MPa maximum stress. K_{tz} is thus calculated to be **2.45**, which validates Roarks empirical solution. In the next step, nonlinear FE analyses were conducted. The material stress-strain curves as determined through the tensile tests and shown in Figure 4 were input in tabular format and a quarter of the punched tensile specimen was meshed. The model consisted of 3350 QUADR elements, which produced 21200 degrees of freedom. The loading occurred in 10 steps, unloading in 4.

The resulting stresses during loading and after unloading are summarized in Figures 9 and 10. In order to compare the results, the nominal stresses were compared for three given strains for each specimen. Aditionally, the residual strain after the largest load cycle was simulated through FEA and compared:

Strain [‰]	Nominal Stress [MPa], Strain [‰]	Nominal Stress Neuber [MPa]	Deviation Neuber [%]	Nominal Stress Mod. Neuber, [MPa]	Deviation Modified Neuber [%]	Nominal Stress FEA [MPa], Strain [‰]	Deviation FEA [%]
2	165.3	171.71	**3.9**	171.71	**3.9**	176.25	**6.6**
3	247.3	237.74	**-3.9**	243.01	**-1.7**	255.84	**3.5**
5	371.5	312.64	**-15.8**	335.01	**-9.8**	343.31	**-7.6**
Residual Strain	0.716	-	-	-	-	-	**193**

Figure 9: Result comparison for punched steel specimen

Strain [‰]	Nominal Stress [MPa], Strain [‰]	Nominal Stress Neuber [MPa]	Deviation Neuber [%]	Nominal Stress Mod. Neuber, [MPa]	Deviation Modified Neuber [%]	Nominal Stress FEA [MPa], Strain [‰]	Deviation FEA [%]
3	80.2	81.6	**1.8**	81.6	**1.8**	83.9	**4.6**
7	158.3	139.1	**-12.1**	147.3	**-6.9**	174.8	**10.4**
12	202.3	182.4	**-9.9**	198.1	**-2.1**	211.1	**4.3**
Residual Strain	5	-	-	-	-	22.0	**340.0**

Figure 10: Result comparison for punched aluminum specimen

The deviation of the approximation methods by Neuber varies between 1.7 and 15.8%. The deviation of the FE analysis ranges from 3.5 to 10.4% compared to the measured data. The residual stresses are over predicted by the FEA analysis by a factor of 1.9 to 3.4.

Figures 11 and 12 graphically show the results. In the upper half the material stress-strain curve is shown along with the Neuber hyperbolas exemplary for two nominal stresses. In the lower half, the specimens yield curve based on the nominal stress is compared for the various methods.

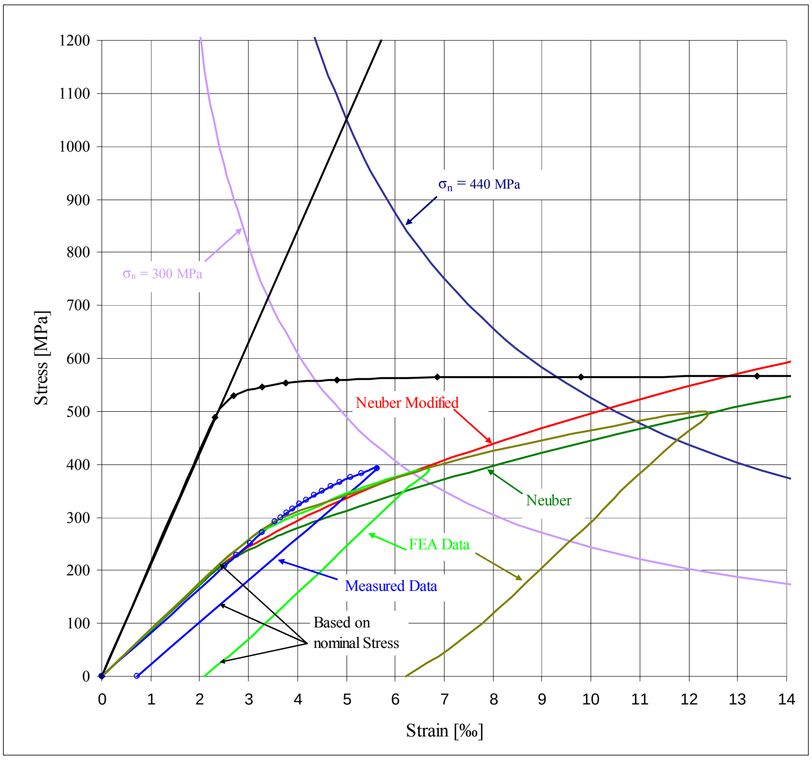

Figure 11: Neuber Hyperbolas and yield curves for the steel specimen

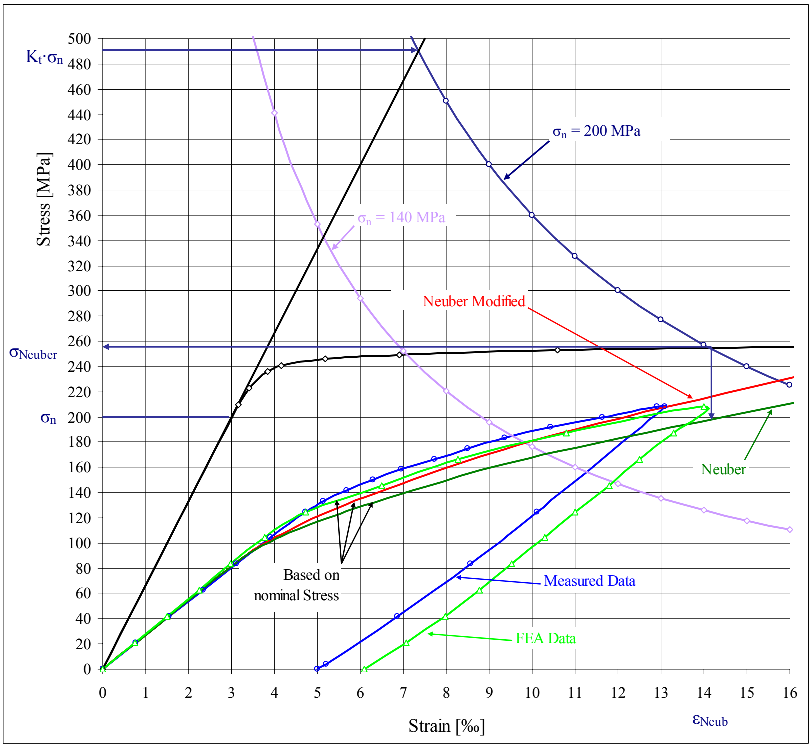

Figure 12: Neuber Hyperbolas and yield curves for the aluminum specimen

5: Conclusions

The approximation of the plastic stress-strain state by utilizing the Neuber formula shows a sufficient correlation for typical engineering use. Nonlinear FE analysis also showed to be generally sufficiently correlating with the measured data. However it has to be investigated why the residual strains are over predicted. Especially Figure 11 shows that the modified Neuber formula becomes invalid past 1.3% local strain. Hence the maximum applicable strains for this approximation are expected to give reasonable results of ~1% maximum strain. While this investigation showed correlation with a uniaxial stress-strain state, the correlation in multiaxial states is not verified.

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